RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2014

SECOND YEAR

STATISTICS (General)

Paper : III

Date : 22/12/2014

Time : 11 am – 1 pm

Full Marks : 50

[Use a separate Answer Book for each group]

Group – A

(*Answer any two questions from Q.No. 1 - 4*) [2×5]

1. a) Pick out the correct alternatives : [1]

   i) Sampling distribution is the distribution of :

   Parameter ; Statistics ; Sample ; Population

   ii) In random sampling with replacement from a population with standard duration σ, if the sample size is equal to the population size (=N), then the standard error of the sample mean will be :

   0 ; σ ; $\sigma/\sqrt{N}$; None

   b) The mean μ of a certain population is equal to the standard error of mean of random samples of size 100 from that population. Find out the standard error of the mean of random samples of size 36 from that population in term of μ [assume that the sampling is with replacement] [2]

   c) Given $F_{8,4,0\cdot05} = 6\cdot04$; Find the value of $F_{4,8,0\cdot95}$. [1]

   d) Express a 't' distribution with n degrees of freedom in terms of F distribution with appropriate degrees of freedom. [1]

2. The value of a population mean increases linearly through time : $\mu(t) = \alpha + \beta t$, while the variance remains constant. Independent simple random samples of size 'n' are taken at times t = 1,2,3.

   Find the conditions on w_1, w_2 & w_3 such that $\hat{\beta} = w_1\bar{x}_1 + w_2\bar{x}_2 + w_3\bar{x}_3$ is an unbiased estimate of the rate of change, β. [$\bar{x}_1, \bar{x}_2, \bar{x}_3$ denotes the sample means at t = 1, 2 & 3 respectively] [5]

3. For a Normal (μ, σ^2) population with $\sigma^2 = 9$, construct a 95% confidence interval for the parametric function $r(\mu) = 2\mu + 7$, on the basis of the random sample of size 25, giving a sample mean of 30. [Given $\Phi(1\cdot96) = 0\cdot975$] [5]

4. What is the level of significance? What is the power of a test? In a testing of hypothesis problem we encounter two types of error —Type 1 error and Type 2 error. Which one is more severe? Can we minimize both? If yes how and if not why? [1+1+1+2]

(*Answer any two questions from Q.No. 5 - 8*) [2×10]

5. a) A random sample $x_1, x_2, ..., x_n$ of size n is drawn from an infinite population with unknown mean μ & unknown variance σ^2. Show that the sample variance $S^2 = \frac{1}{n}\sum(x_i - \bar{x})^2$ is not an unbiased estimator of σ^2. Find the unbiased estimator of σ^2. [3]

   b) Find the maximum likelihood estimators of a the parameters of a $N(\mu, \sigma^2)$ population on the basis of the random sample $(x_1, x_2, ..., x_n)$ of size n when—

   - population Variance is given $= \sigma_0^2$ (say)
   - population mean μ is known = μ_0 (say)
   - both the parameters are known. [7]

6. a) Consider a $N(\mu,\sigma^2)$ population where both μ & σ^2 are unknown. We are to test the null hypothesis $H_0:\mu=\mu_0$ against the alternative $H_{1A}:\mu>\mu_0$; $H_{1B}:\mu<\mu_0$; $H_{1C}:\mu\neq\mu_0$.

   On the basis of the random sample (x_1, x_2, ...,x_n) of size n, construct the critical regions (of size α) corresponding to the three alternatives. [5]

   b) Let p be the probability that a coin will fall head. To test $H_0:p=\frac{1}{2}$ ag $H_1:p=\frac{3}{4}$; a coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type I error and power of the test. [5]

7. a) What is a Pearsonian χ^2 -statistic? [2]

   b) In statistical inference, explain the application of Pearsonian statistics for testing—
   - Goodness of fit
   - Independence of two attributes [8]

8. a) If X_1, X_2, X_3 be random samples from $N(0,\sigma^2)$ population, what is the distribution of $(X_1^2+X_2^2+X_3^2)/\sigma^2$? State (without derivation) the sampling distribution of the statistics $\sqrt{2}X_1/\sqrt{X_2^2+X_3^2}$ & X_1^2/X_2^2, mentioning the appropriate degrees of freedom? [3]

   b) Consider, for a bivariate normal population, testing the null hypothesis $H_0:\rho=0$ against $H_i:\rho\neq0$, where ρ is the population correlation coefficient. The test is being performed on the basis of 18 pairs of observations from the bivarate normal population. What is the least value of r (sample Correlation Coefficient) for which the null hypothesis will be rejected at 5% level of significance? [Given, $t_{0\cdot025,16}=2$] [7]

<u>Group – B</u>

(*Answer <u>any two</u> questions from <u>Q.No. 9 - 12</u>*) [2×5]

9. Define a Timeseries. Mention its important components with illustrations. Also indicate its importance in business and economics.

10. a) What are the different types of models in Time Series? [2]

    b) Explain Cylical and Irregular component. [3]

11. Define the following Index Numbers and discuss their merits and demerits :
    a) Laspeyres' Index Number
    b) Paasche's Index Number
    c) Fisher's Ideal Index Number

12. What is Time Reversal Test? Out of Laspeyres', Paasche's and Fisher's Index, which passes Time Reversal Test and how? [1+4]

(*Answer either <u>Q.No. 13 or 14</u>*) [1×10]

13. Describe briefly the various problems that are involved in the construction of an index number of prices.

    Examine Edgeworth Marshall's Price Index Number in the light of the two common tests of an Index Number.

14. Describe : a) the Least Square Method

    b) the Moving average method for the determination of trend in a time series.

    Mention the merits and demerits of the above methods.

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